C4 **SERIES**   *Worksheet C*

1 **a** Expand $(1 - 4x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 and state
the set of values of x for which the expansion is valid. **(4)**

   b By substituting $x = 0.01$ in your expansion, find the value of $\sqrt{6}$ to 6 significant figures. **(3)**

2 $f(x) \equiv \dfrac{4}{1 + 2x - 3x^2}$.

   a Express $f(x)$ in partial fractions. **(3)**

   b Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x up to and
including the term in x^3 and state the set of values of x for which the expansion is valid. **(5)**

3 **a** Expand $(2 - x)^{-2}$, $|x| < 2$, in ascending powers of x up to and including the term in x^3. **(4)**

   b Hence, find the coefficient of x^3 in the series expansion of $\dfrac{3 - x}{(2 - x)^2}$. **(2)**

4 $f(x) \equiv \dfrac{4}{\sqrt{1 + \frac{2}{3}x}}$, $-\frac{3}{2} < x < \frac{3}{2}$.

   a Show that $f(\frac{1}{10}) = \sqrt{15}$. **(2)**

   b Expand $f(x)$ in ascending powers of x up to and including the term in x^2. **(3)**

   c Use your expansion to obtain an approximation for $\sqrt{15}$, giving your answer as an
exact, simplified fraction. **(2)**

   d Show that $3\frac{55}{63}$ is a more accurate approximation for $\sqrt{15}$. **(2)**

5 **a** Expand $(1 - x)^{\frac{1}{3}}$, $|x| < 1$, in ascending powers of x up to and including the term in x^2. **(3)**

   b By substituting $x = 10^{-3}$ in your expansion, find the cube root of 37 correct to
9 significant figures. **(3)**

6 The series expansion of $(1 + 5x)^{\frac{3}{5}}$, in ascending powers of x up to and including the
term in x^3, is
$$1 + 3x + px^2 + qx^3, \ |x| < \tfrac{1}{5} .$$

   a Find the values of the constants p and q. **(4)**

   b Use the expansion with a suitable value of x to find an approximate value for $(1.1)^{\frac{3}{5}}$. **(2)**

   c Obtain the value of $(1.1)^{\frac{3}{5}}$ from your calculator and hence find the percentage error in
your answer to part **b**. **(2)**

7 **a** Find the values of A, B and C such that
$$\frac{8 - 6x^2}{(1 + x)(2 + x)^2} \equiv \frac{A}{1 + x} + \frac{B}{2 + x} + \frac{C}{(2 + x)^2} .$$ **(4)**

   b Hence find the series expansion of $\dfrac{8 - 6x^2}{(1 + x)(2 + x)^2}$, $|x| < 1$, in ascending powers of x up

to and including the term in x^3, simplifying each coefficient. **(7)**

8   **a** Expand $(1 - 2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^2.   **(3)**

    b By substituting $x = 0.0008$ in your expansion, find the square root of 39 correct to
    7 significant figures.   **(4)**

9   **a** Find the series expansion of $(1 + 8x)^{\frac{1}{3}}$, $|x| < \frac{1}{8}$, in ascending powers of x up to and
    including the term in x^2, simplifying each term.   **(3)**

    b Find the exact fraction k such that

$$\sqrt[3]{5} = k\sqrt[3]{1.08}$$ **(2)**

    c Hence, use your answer to part **a** together with a suitable value of x to obtain an
    estimate for $\sqrt[3]{5}$, giving your answer to 4 significant figures.   **(3)**

10 $$f(x) \equiv \frac{6x}{x^2 - 4x + 3}, \quad |x| < 1.$$

    a Express f(x) in partial fractions.   **(3)**

    b Show that for small values of x,

$$f(x) \approx 2x + \tfrac{8}{3}x^2 + \tfrac{26}{9}x^3.$$ **(5)**

11   **a** Find the binomial expansion of $(4 + x)^{\frac{1}{2}}$ in ascending powers of x up to and including
    the term in x^2 and state the set of values of x for which the expansion is valid.   **(4)**

    b By substituting $x = \frac{1}{20}$ in your expansion, find an estimate for $\sqrt{5}$, giving your
    answer to 9 significant figures.   **(3)**

    c Obtain the value of $\sqrt{5}$ from your calculator and hence comment on the accuracy of
    the estimate found in part **b**.   **(2)**

12   **a** Expand $(1 + 2x)^{-\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3.   **(4)**

    b Hence, show that for small values of x,

$$\frac{2 - 5x}{\sqrt{1 + 2x}} \approx 2 - 7x + 8x^2 - \tfrac{25}{2}x^3.$$ **(3)**

    c Solve the equation

$$\frac{2 - 5x}{\sqrt{1 + 2x}} = \sqrt{3}.$$ **(3)**

    d Use your answers to parts **b** and **c** to find an approximate value for $\sqrt{3}$.   **(2)**

13   **a** Expand $(1 + x)^{-1}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3.   **(2)**

    b Hence, write down the first four terms in the expansion in ascending powers of x
    of $(1 + bx)^{-1}$, where b is a constant, for $|bx| < 1$.   **(1)**

    Given that in the series expansion of

$$\frac{1 + ax}{1 + bx}, \quad |bx| < 1,$$

    the coefficient of x is -4 and the coefficient of x^2 is 12,

    c find the values of the constants a and b,   **(5)**

    d find the coefficient of x^3 in the expansion.   **(2)**

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